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Matrix Regularization of Open Supermembrane

—towards M-theory five-brane via open supermembrane —

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Abstract

We study open supermembranes in 11 dimensional rigid superspace with 6 dimensional topological defects (M-theory five-branes). After rederiving in the Green-Schwarz formalism the boundary conditions for open superstrings in the type IIA theory, we determine the boundary conditions for open supermembranes by imposing kappa symmetry and invariance under a fraction of 11 dimensional supersymmetry. The result seems to imply the self-duality of the three-form field strength on the five-brane world volume. We show that the light-cone gauge formulation is regularized by a dimensional reduction of a 6 dimensional N=1 super Yang-Mills theory with the gauge group SO(N$\to\infty$). We also analyze the SUSY algebra and BPS states in the light-cone gauge.

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1 Introduction

During the recent progress in the understanding of nonperturbative properties of superstring theory, M-theory has played an indispensable role. While its formulation is still controversial, it is widely believed to possess the following properties:

1. the effective low energy theory is described by 11 dimensional supergravity[1];
2. the compactification on S^1 coincides with the type IIA superstring theory[1];
3. the compactification on $S^1/\mathbf{Z}_2$ yields the $E_8 \times E_8$ heterotic string theory[2];
4. membranes and five-branes play central roles [3].

Let us focus on the five-branes. While covariant formulations of M-theory five-branes have been found recently [4] [5], it is still important to look for their alternative description. In the case of superstring theory, solitonic objects carrying Ramond-Ramond charges are described as Dirichlet branes on which open strings can end[6]. This leads us to the idea of describing M-theory five-branes as "Dirichlet-branes" on which open supermembranes can end [7]. Becker and Becker [8] identified boundary conditions for an open supermembrane in a particular kind of light-cone gauge.

In this paper we make a further step towards the description of five-branes by means of open supermembranes. In sec.II we investigate open supermembranes in the 11 dimensional rigid superspace in the covariant formalism[9]. It is known that an open supermembrane cannot exist without the topological defects on which the membrane can end [9][10]. We reexamine the analysis of ref.[10] and show that the topological defects should be of dimension 2 (string), 6 (five-brane) or 10 (nine-brane). We determine the boundary conditions for the open supermembrane in the presence of a three-form field strength on the five-brane world volume by requiring the invariance of the action under kappa-symmetry and 1/2 supersymmetry. The result seems to imply that the three-form field strength be self-dual. As an exercise before the analysis of open supermembranes, we investigate, in the covariant Green-Schwarz formalism[11], open superstrings in the type IIA theory in the presence of a field strength on the D brane world volume. We rederive the boundary conditions found in the light-cone gauge[12]. In sec.III, we construct the light-cone gauge formalism of the open supermembrane by mimicking the procedure for the closed supermembrane[13]. The resulting theory is a (0+1)-dimensional $N = 8$ supersymmetric gauge theory with gauge group being that of area preserving diffeomorphisms (APD) which preserve the boundary conditions. We show that the theory is well-approximated by a dimensional reduction to (0+1)-dimension of a (5+1)-dimensional N=1 SO(N) super Yang-Mills theory with a hypermultiplet in the rank-2 symmetric tensor representation of SO(N). In this SO(N) regularization, however, correspondence between surface integral and trace is not so complete as in the U(N) regularization of the closed membrane. Thus in subsec.III C we propose an alternative regularization of the integration based on the idea of a "non-commutative cylinder". Sec.IV is devoted to the analysis of the 11D SUSY algebra and that of BPS states in the light-cone gauge. In sec.V we discuss possible extensions of our results and some remaining issues.

In Appendix A, the reader can find the convention and formulae used in this paper.

2 Boundary conditions for open supermembranes

Boundary conditions for open supermembranes were analyzed in refs.[8][10] when the background is the 11 dimensional rigid superspace.[1] In this section we extend the analysis to the case of a nonzero three-form field strength on the world volume of a "Dirichlet brane" by using the covariant formalism of the supermembrane theory [9]. We argue that the field strength seems to be identified with that of the two-form potential on the five-brane when the "Dirichlet brane" is 6 dimensional. In order to get accustomed to its treatment, we begin by examining boundary conditions for open superstrings in the Green-Schwarz (GS) formalism of the type IIA theory when there is a constant two-form field strength on the D-brane world volume. The key to determine the boundary condition resides in the kappa-symmetry and supersymmetry.

2.1 Open superstrings in the type IIA theory

Our starting point is the Green-Schwarz action of the type IIA superstring[11]

$$\begin{aligned} S &= \int_\Sigma d^2\sigma\sqrt{-g} + \int_\Sigma \mathcal{L}_{WZ} + \int_{\partial\Sigma} A, \\ \mathcal{L}_{WZ} &= -i\bar{\theta}\Gamma_\mu\Gamma_{11}d\theta(dX^\mu - \frac{i}{2}\bar{\theta}\Gamma^\mu d\theta), \end{aligned} \tag{1}$$

where $(X^\mu(\sigma), \theta^\alpha(\sigma))$ $(\mu = 0, 1, \ldots, 9; \alpha = 1, 2, \ldots, 32)$ denotes the embedding of the string world sheet Σ (with boundary $\partial\Sigma$) into the 10 dimensional type IIA rigid superspace. We note that θ is a Majorana spinor with $\bar{\theta} = \theta^T\mathcal{C}$. $A = dX^\mu A_\mu + d\theta^\alpha A_\alpha$ is a one-form on the world volume of the Dirichlet-brane.[2] For simplicity we consider only the case that the one-form is bosonic, i.e.,

$$A_\alpha = 0, \quad \partial_\alpha A_\mu = 0.$$

g denotes the determinant of the induced metric on the world sheet

$$\begin{aligned} g_{rs} &= \Pi_r^\mu \Pi_s^\nu \eta_{\mu\nu} \quad (r, s = \tau, \sigma), \\ \Pi_r^\mu &= \partial_r X^\mu - i\bar{\theta}\Gamma^\mu\partial_r\theta. \end{aligned} \tag{2}$$

In the case of the closed superstring, the action (1) has symmetry under the 10 D type IIA super Poincaré transformations, world sheet reparametrizations and local fermionic transformations (kappa-symmetry). Among them we only give the fermionic transformations:

$$\delta_\epsilon X^\mu = -i\bar{\theta}\Gamma^\mu\epsilon, \quad \delta_\epsilon\theta = \epsilon, \tag{3}$$

$$\delta_\kappa X^\mu = i\bar{\theta}\Gamma^\mu(1+\Gamma)\kappa, \quad \delta_\kappa\theta = (1+\Gamma)\kappa, \tag{4}$$

[1] Bosonic open membranes were studied in ref.[14] and a preliminary investigation of an open supermembrane was made in ref.[9].

[2] As for differential forms we use the convention of Wess-Bagger [15]. In this paper we do not make explicit distinction between the differential form on the superspace (or on the Dirichlet brane) and its pullback to the world sheet, because which is used is usually obvious from the context.

where ϵ is a constant 10D Majorana spinor and κ is a 10D Majorana spinor which depends on the coordinates of the world sheet. The matrix Γ is defined as

$$\Gamma = \frac{\epsilon^{rs}}{2\sqrt{-g}} \Pi_r^\mu \Pi_s^\nu \Gamma_{\mu\nu} \Gamma_{11}, \tag{5}$$

and is subject to

$$(\Gamma)^2 = I_{32}, \quad \mathcal{C}^{-1}\Gamma^T \mathcal{C} = \Gamma.$$

The kappa-symmetry is particularly important in the GS formalism because it, together with the world sheet reparametrization invariance, guarantees the matching of bosonic and fermionic degrees of freedom on the world sheet. Thus, in order to preserve a fraction of supersymmetry, we have to keep the kappa-symmetry even in the presence of the world-sheet boundary $\partial\Sigma$. In the following we will look for the boundary conditions which preserve kappa-symmetry and 1/2 of space-time SUSY.

First we investigate the kappa-symmetry. Because it is preserved in the absence of $\partial\Sigma$, when we take the variation of the action under the local fermionic transformation we are left only with the boundary terms

$$\begin{aligned} \delta_\kappa S &= \int_{\partial\Sigma} \left[\frac{1}{2} (\bar\theta \Gamma_\mu \Gamma_{11} d\theta \bar\theta \Gamma^\mu \delta_\kappa \theta + \bar\theta \Gamma_\mu \Gamma_{11} \delta_\kappa \theta \bar\theta \Gamma^\mu d\theta) + i\bar\theta \Gamma_\mu \Gamma_{11} \delta_\kappa \theta dX^\mu \right] \\ &\quad + \int_{\partial\Sigma} dX^\mu \delta_\kappa X^\nu F_{\nu\mu}, \end{aligned} \tag{6}$$

where $F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}$ is the two-form field strength. These boundary terms vanish if the following conditions hold on $\partial\Sigma$:

$$\begin{aligned} dX^{\bar a} &= \bar\theta \Gamma^{\bar a} d\theta = \bar\theta \Gamma^{\bar a} (1+\Gamma)\kappa = 0, \\ \bar\theta \Gamma_{\underline{\mu}} \Gamma_{11} (1+\Gamma)\kappa &= F_{\underline{\mu}\underline{\nu}} \bar\theta \Gamma^{\underline{\nu}} (1+\Gamma)\kappa, \end{aligned} \tag{7}$$

where $\underline{\mu}, \underline{\nu} = 0, 1, \cdots, p$ and $\bar a = p+1, \cdots, 9$.

Next we see the conditions for unbroken SUSY. As in the case of kappa-symmetry we find

$$\begin{aligned} \delta_\epsilon S &= \int_{\partial\Sigma} \left[-\frac{1}{6} (\bar\theta \Gamma_\mu \Gamma_{11} \epsilon \bar\theta \Gamma^\mu d\theta + \bar\theta \Gamma^\mu \epsilon \bar\theta \Gamma_\mu \Gamma_{11} d\theta) - i\bar\theta \Gamma_\mu \Gamma_{11} \epsilon dX^\mu \right] \\ &\quad + \int_{\partial\Sigma} dX^\mu (-i\bar\theta \Gamma^\nu \epsilon) F_{\nu\mu}. \end{aligned} \tag{8}$$

It vanishes if we set, on $\partial\Sigma$,

$$\begin{aligned} dX^{\bar a} &= \bar\theta \Gamma^{\bar a} d\theta = \bar\theta \Gamma^{\bar a} \epsilon = 0, \\ \bar\theta \Gamma_{\underline{\mu}} \Gamma_{11} \epsilon &= F_{\underline{\mu}\underline{\nu}} \bar\theta \Gamma^{\underline{\nu}} \epsilon. \end{aligned} \tag{9}$$

We have derived eqs.(7) and (9) from the kappa-symmetry and supersymmetry respectively. These two equations are of the same form except that $(1+\Gamma)\kappa$ in the former is replaced with ϵ in the latter. Thus we find it natural to impose the following boundary conditions on $\partial\Sigma$:

$$\begin{aligned} \delta X^{\bar a} &= \bar\theta \Gamma^{\bar a} \delta\theta = 0 \quad (\bar a = p+1, \ldots, 9), \\ \bar\theta \Gamma_{\underline{\mu}} \Gamma_{11} \delta\theta &= F_{\underline{\mu}\underline{\nu}} \bar\theta \Gamma^{\underline{\nu}} \delta\theta \qquad (\underline{\mu} = 0, 1, \cdots, p). \end{aligned} \tag{10}$$

This represents the situation in which the open superstring ends on a (p+1)-dimensional hyperplane, namely, on a D p-brane.

In order that a fraction of space-time SUSY be unbroken, however, it is necessary to rewrite the boundary conditions for the fermion θ in a linear form. This requirement allows only the even integer p. In this case the above boundary conditions are rewritten as

$$\begin{aligned} X^{\bar{a}} &= \text{const.}\ , \\ \theta &= e^{\frac{1}{2}Y_{\underline{\mu\nu}}\Gamma^{\underline{\mu\nu}}\Gamma_{11}}(\Gamma_{11})^{\frac{p-2}{2}}\Gamma_{(p)}\theta \\ &= e^{\frac{1}{4}Y_{\underline{\mu\nu}}\Gamma^{\underline{\mu\nu}}\Gamma_{11}}(\Gamma_{11})^{\frac{p-2}{2}}\Gamma_{(p)}e^{-\frac{1}{4}Y_{\underline{\mu\nu}}\Gamma^{\underline{\mu\nu}}\Gamma_{11}}\theta, \end{aligned} \tag{11}$$

where $F_{\underline{\mu}}{}^{\underline{\nu}} = \tanh(Y)_{\underline{\mu}}{}^{\underline{\nu}}$ should be constant so that (11) yields (10). This result coincides with that obtained in ref. [12] (see also ref.[16]).

Boundary conditions for the remaining fields, namely $X^{\underline{\mu}}$ and

$$\theta^{(+)} \equiv \frac{1}{2}(1 + e^{\frac{1}{2}Y_{\underline{\mu\nu}}\Gamma^{\underline{\mu\nu}}\Gamma_{11}}(\Gamma_{11})^{\frac{p-2}{2}}\Gamma_{(p))})\theta,$$

are determined from the compatibility with equations of motion. Namely, in deriving equations of motion from the variations of the action, we should choose boundary conditions such that boundary terms vanish. This leads us to find

$$\Phi^{\underline{\mu}} \equiv \sqrt{-g}g^{\sigma s}\Pi_s^{\underline{\mu}} - F^{\underline{\mu}}{}_{\underline{\nu}}\Pi_\tau^{\underline{\nu}} = 0. \tag{12}$$

In principle we can specify the boundary conditions completely by exploiting (12), its compatibility with kappa-symmetry: $\delta_\kappa \Phi^{\underline{\mu}} = 0$, and the equation of motion for θ:

$$\Pi_r^\mu \Gamma_\mu(\sqrt{-g}g^{rs}\partial_s + \epsilon^{rs}\Gamma_{11}\partial_s)\theta = 0. \tag{13}$$

In general, however, it is difficult to carry out this task because the conditions are fairly non-linear. We therefore restrict ourselves to the following two cases which are relatively tractable.

(1)$\underline{F_{\underline{\mu\nu}} = 0}$. In this case we can reduce the condition (12) to the linear one

$$\Pi_\sigma^{\underline{\mu}} = 0. \tag{14}$$

If we impose the remaining two conditions we can separate the bosonic and the fermionic parts as

$$\partial_\sigma X^{\underline{\mu}} = \partial_\sigma \theta^{(+)} = 0 \quad \text{on } \partial\Sigma. \tag{15}$$

(2)<u>Light-cone conformal gauge</u>. In this gauge

$$X^+ = \tau, \quad \Gamma^+\theta = 0, \quad g_{rs} \propto \delta_{rs},$$

eq.(12) is simplified as

$$\partial_\sigma X^{\underline{a}} = F_{\underline{ab}}\partial_\tau X^{\underline{b}} + F_{\underline{a}+}, \tag{16}$$

where $X^\pm = \frac{1}{\sqrt{2}}(X^1 \pm X^0)$, and $\underline{a} = 2, \cdots, p$. In order for eq.(12) to be compatible with the light-cone gauge, we must have $F^{+\underline{\mu}} = F_{-\underline{\mu}} = 0$. In the light-cone gauge, the kappa-symmetry is gauge-fixed and the equation of motion is simplified as $\partial_\tau\theta = \Gamma_{11}\partial_\sigma\theta$. Thus we find

$$[1 + e^{\frac{1}{2}Y_{\underline{ab}}\Gamma^{\underline{ab}}\Gamma_{11}}(\Gamma_{11})^{\frac{p-2}{2}}\Gamma_{(p)}]\partial_\sigma\theta = 0 \quad \text{on } \partial\Sigma. \tag{17}$$

We note that the compatibility with supersymmetry further requires $F_{+\underline{a}}$ to vanish.

2.2 Open supermembranes

Let us now investigate open supermembranes. We consider the case in which a two-form gauge field $B = \frac{1}{2} dX^\mu dX^\nu B_{\nu\mu}$ couples to the boundary of the membrane world volume.[3] The relevant action is

$$
\begin{aligned}
S &= -\int_\Sigma d^3\xi \sqrt{-g} + \int_\Sigma \mathcal{L}_{WZ} + \int_{\partial\Sigma} B, \\
\mathcal{L}_{WZ} &= \frac{i}{2}\bar\theta\Gamma_{\mu\nu}d\theta\left[(dX^\mu - i\bar\theta\Gamma^\mu d\theta)dX^\nu - \frac{1}{3}\bar\theta\Gamma^\mu d\theta\bar\theta\Gamma^\nu d\theta\right], \qquad (18)
\end{aligned}
$$

where $(X^\mu(\xi), \theta^\alpha(\xi))$ $(\mu = 0, 1, \ldots, 10;\ \alpha = 1, 2, \ldots, 32)$ denotes the embedding of the membrane world volume Σ (with boundary $\partial\Sigma$)[4] into the 11 dimensional rigid superspace. As in the case of type IIA strings θ is a Majorana spinor with $\bar\theta = \theta^T \mathcal{C}$. We mean by g the determinant of the induced metric

$$
\begin{aligned}
g_{ij} &= \Pi_i^\mu \Pi_j^\nu \eta_{\mu\nu}, \quad (i, j = 0, 1, 2) \\
\Pi_i^\mu &= \partial_i X^\mu - i\bar\theta\Gamma^\mu\partial_i\theta. \qquad (19)
\end{aligned}
$$

In the case of a closed supermembrane, the action (18) is invariant under 11D super Poincaré transformations, world volume reparametrizations, and local fermionic transformations (kappa-symmetry). We only give expressions for fermionic transformations

$$
\begin{aligned}
\delta_\epsilon X^\mu &= -i\bar\theta\Gamma^\mu\epsilon, \quad \delta_\epsilon\theta = \epsilon, \\
\delta_\kappa X^\mu &= i\bar\theta\Gamma^\mu(1+\Gamma)\kappa, \quad \delta_\kappa\theta = (1+\Gamma)\kappa, \qquad (20)
\end{aligned}
$$

where ϵ is a constant 11D Majorana spinor and κ is a 11D Majorana spinor which depends on (ξ^i). The matrix Γ is defined as

$$
\Gamma = \frac{\epsilon^{ijk}}{3!\sqrt{-g}} \Pi_i^\mu \Pi_j^\nu \Pi_k^\rho \Gamma_{\mu\nu\rho}, \qquad (21)
$$

and has the following properties

$$
(\Gamma)^2 = I_{32}, \quad \mathcal{C}^{-1}\Gamma^T\mathcal{C} = \Gamma, \quad \Pi_i^\mu\Gamma\Gamma_\mu = \Pi_i^\mu\Gamma_\mu\Gamma = \frac{g_{im}}{2\sqrt{-g}}\epsilon^{mkl}\Pi_k^\nu\Pi_l^\rho\Gamma_{\nu\rho}. \qquad (22)
$$

[3] In general we can consider a coupling $S_{int} = -\int_\Sigma C$ of the membrane world volume to the three-form potential C which is a member of the 11 dimensional supergravity multiplet. The two-form $B^{(0)}$ is introduced in order to maintain the gauge invariance of the theory[10]. Actually $S + S_{int}$ (with B in (18) replaced with $B^{(0)}$) is invariant under the gauge transformations $\delta C = d\Lambda$ and $\delta B^{(0)} = \Lambda$, where Λ is a space-time two-form field.

Because we are now working in the 11 dimensional rigid superspace in which $dC = 0$ (see e.g. [9]), we can express the three-form by a pure gauge, $C = d\Lambda^{(0)}$, and thus we can absorb it into the two-form $B^{(0)}$.

From this consideration, we see that the two-form field B in eq.(18) should actually be regarded as the gauge-invariant object $B^{(0)} - \Lambda^{(0)}$ and that the three-form field strength $H = dB$ equals $dB^{(0)} - C$ which coincides with the gauge invariant field strength introduced in[7].

[4] We use the world-volume coordinate $(\xi^i) = (\tau, \sigma^1, \sigma^2)$, among which (τ, σ^2) and σ^1 are, respectively, tangent and normal to the boundary $\partial\Sigma$. As for the volume form we use the convention $d\xi^i d\xi^j d\xi^k = \epsilon^{ijk} d^3\xi$.

As in the string case kappa-symmetry is indispensable if we want to keep a part of world volume supersymmetry. In the following we determine the boundary conditions by imposing the invariance under kappa-symmetry and under a fraction of 11D SUSY. Variations of the action (18) under the transformations (20) are computed as

$$\begin{aligned}
\delta_\kappa S &= \int_{\partial\Sigma}\Big[\frac{i}{2}\bar\theta\Gamma_{\mu\nu}d\theta(idX^\mu\bar\theta\Gamma^\nu\delta_\kappa\theta+\frac{1}{3}\bar\theta\Gamma^\mu d\theta\bar\theta\Gamma^\nu\delta_\kappa\theta) \\
&+\frac{i}{2}\bar\theta\Gamma_{\mu\nu}\delta_\kappa\theta(dX^\mu dX^\nu - i\bar\theta\Gamma^\mu d\theta dX^\nu - \frac{1}{3}\bar\theta\Gamma^\mu d\theta\bar\theta\Gamma^\nu d\theta)\Big] \\
&+\int_{\partial\Sigma}(-\frac{i}{2}dX^\mu dX^\nu H_{\mu\nu\rho}\bar\theta\Gamma^\rho\delta_\kappa\theta), \\
\delta_\epsilon S &= \int_{\partial\Sigma}\Big[-\frac{i}{2}\bar\theta\Gamma_{\mu\nu}\epsilon(dX^\mu dX^\nu - \frac{i}{3}\bar\theta\Gamma^\mu d\theta dX^\nu - \frac{1}{15}\bar\theta\Gamma^\mu d\theta\bar\theta\Gamma^\nu d\theta) \\
&+\frac{1}{6}\bar\theta\Gamma^\nu\epsilon\bar\theta\Gamma_{\mu\nu}d\theta(dX^\mu - \frac{i}{5}\bar\theta\Gamma^\mu d\theta)\Big] \\
&+\int_{\partial\Sigma}\frac{i}{2}dX^\mu dX^\nu H_{\mu\nu\rho}\bar\theta\Gamma^\rho\epsilon, \qquad (23)
\end{aligned}$$

where $H = dB$ is the three-form field strength. In order for $\delta_\kappa S$ and $\delta_\epsilon S$ to vanish it is sufficient to set up the following boundary conditions on $\partial\Sigma$:

$$\begin{aligned}
\delta X^{\bar a} &= \bar\theta\Gamma^{\bar a}\delta\theta = 0, \\
\bar\theta\Gamma_{\underline{\mu}\underline{\nu}}\delta\theta &= H_{\underline{\mu}\underline{\nu}\underline{\rho}}\bar\theta\Gamma^{\underline{\rho}}\delta\theta, \qquad (24)
\end{aligned}$$

where $\underline{\mu} = 0, 1, \ldots, p$ and $\bar a = p+1, \ldots, 10$. The first equation represents the situation in which the open supermembrane ends on a (p+1)-dimensional hyperplane-like topological defect in the rigid 11D superspace. However, this is not the whole story. In order to keep a fraction of 11D supersymmetry, the boundary conditions for θ have to be rewritten in a linear form. From the upper equation of (24) we can infer a natural candidate for the desired linear boundary condition

$$\begin{aligned}
\theta &= F(\Gamma^{\underline{\mu}}; H_{\underline{\mu}\underline{\nu}\underline{\rho}})\Gamma_{(p)}\theta, \quad F(\Gamma^{\underline{\mu}}; 0) = I_{32}, \\
I_{32} &= (F(\Gamma^{\underline{\mu}}; H_{\underline{\mu}\underline{\nu}\underline{\rho}})\Gamma_{(p)})^2. \qquad (25)
\end{aligned}$$

The third equation follows from the consistency of the first equation. Note that $F(\Gamma^{\underline{\mu}}; H_{\underline{\mu}\underline{\nu}\underline{\rho}})$ must be real (in the Majorana representation) because θ is a Majorana spinor.

We first consider the $H_{\underline{\mu}\underline{\nu}\underline{\rho}} = 0$ case. From the consistency condition $(\Gamma_{(p)})^2 = I_{32}$, we find that $\frac{p(p+1)}{2}$ be odd. Moreover, in order to reproduce eq.(24), we have to set p to be odd. It implies that this theory admits only the $(p+1)$ dimensional topological defects with

$$p = 1, 5, 9. \qquad (26)$$

The $p = 5$ case represents the M-theory five-brane and $p = 9$ is related to Hořava-Witten's "end-of-the-world 9-branes" [2]. What is puzzling is the $p = 1$ case. It would be interesting to pursue it further. In this paper, however, we mainly concentrate on the $p = 5$ case.

Let us next consider the case of nonzero $H_{\mu\nu\rho}$. Unlike in the string case the last condition of (24) cannot be interpreted as rotation in the five-brane world volume, and thus it is difficult to find out $F(\Gamma^{\underline{\mu}}; H_{\underline{\mu\nu\rho}})$ which reproduces (24). In a special case in which $H_{\underline{\mu\nu\rho}}$ satisfies a "self-duality" condition, however, we can construct such F. The boundary condition in this case turns out to be

$$\theta = \exp\left(\frac{-1}{3} h_{\underline{\mu\nu\rho}}\Gamma^{\underline{\mu\nu\rho}}\right)\Gamma_{(5)}\theta, \quad h_{\underline{\mu\nu\rho}} = \frac{1}{3!}\epsilon_{\underline{\mu\nu\rho\sigma\kappa\lambda}} h^{\underline{\sigma\kappa\lambda}} (= \text{const.}). \tag{27}$$

After lengthy and tedious calculation which is outlined in Appendix B, we find that eq.(27) reproduces the last condition of eq.(24) provided that

$$H_{\underline{\mu\nu\rho}} = 4h_{\underline{\mu\nu\sigma}}(1-2k)^{-1\underline{\sigma}}{}_{\underline{\rho}}, \tag{28}$$

where $k^{\underline{\mu}}_{\underline{\nu}} = h^{\underline{\mu\rho\sigma}} h_{\underline{\nu\rho\sigma}}$. The pattern of the breakdown of 11D SUSY following from (27) agrees with that obtained from the analysis of the five-brane dynamics[5].[5]

We remark that the derivation of the above result depends heavily on the self-duality of $h_{\underline{\mu\nu\rho}}$. While we have not yet been able to provide a complete proof, we strongly believe that eq.(27) is the unique possibility of the linear boundary condition and that the "self-duality" of $H_{\mu\nu\rho}$ naturally follows from the requirement of kappa-symmetry and space-time SUSY.

The remaining boundary conditions are determined by investigating boundary terms arising from the action principle. After some computation we find the following boundary condition

$$\Phi^{\underline{\mu}} \equiv \sqrt{-g} g^{1k}\Pi^{\underline{\mu}}_k - H^{\underline{\mu}}{}_{\underline{\nu\rho}}\Pi^{\underline{\nu}}_2\Pi^{\underline{\rho}}_\tau = 0. \tag{29}$$

In principle we can completely determine the boundary conditions by further imposing $\delta_\kappa\Phi^{\underline{\mu}} = 0$ and by considering the equation of motion: $\Pi^\mu_i\Gamma_\mu(1-\Gamma)g^{ij}\partial_j\theta = 0$. In the case of nonzero $H_{\underline{\mu\nu\rho}}$, however, the conditions become highly nonlinear and it is difficult to reduce them to a tractable form. From now on we therefore consider the case $H_{\underline{\mu\nu\rho}} = 0$ only. In this case we can set the Neumann boundary conditions

$$\begin{aligned} \partial_1 X^{\underline{\mu}} &= 0, \\ (1+\Gamma_{(p)})\partial_1\theta &= 0. \end{aligned} \tag{30}$$

Before concluding this section we consider the restriction on the world volume reparametrization. Under the infinitesimal reparametrization $\xi^i \to \xi^i + v^i$, world volume fields transform as

$$\delta_v X^\mu = v^i\partial_i X^\mu, \quad \delta_v(\partial_i X^\mu) = v^j\partial_j(\partial_i X^\mu) + \partial_i v^j \partial_j X^\mu,$$

and the resulting variation of the action is

$$\delta_v S = \int_\Sigma d^3\xi \partial_i(v^i\mathcal{L}) = -\int_{\partial\Sigma} d\tau d\sigma^2 v^1 \mathcal{L}.$$

[5] Our $H_{\underline{\mu\nu\rho}}$ seems to correspond to $4e_a{}^m e_b{}^n e_c{}^p H_{mnp}$ in ref.[5]. Actually eq.(28) coincides with eq.(53) of ref.[5] if we take account of this correspondence and the fact that $H = dB$ is rewritten as $dB^{(0)} - C$. Here $B^{(0)}$ is the "bare" two-form field on the five-brane and C is the three-form potential in 11 dimensions (see footnote 3).

Imposing the conditions that $\delta_v S$ vanish and that the boundary conditions (24) (30) be preserved by the above transformation, we find the following boundary conditions for the generator v^i:

$$
\begin{aligned}
v^1 &= 0, \\
\partial_1 v^0 = \partial_1 v^2 &= 0.
\end{aligned}
\tag{31}
$$

3 Matrix regularization of an open supermembrane

The matrix regularization of a closed supermembrane was proposed by de Wit, Hoppe and Nicolai (dWHN)[13]. Following their prescription we construct the matrix regularization of an open supermembrane. For simplicity, we investigate the case in which there exist(s) either one or two parallel five-brane(s). In this situation, only DD and NN sectors appear; therefore we need not consider either DN or ND sector.

3.1 Light-cone gauge formulation

Because the matrix regularization of the dWHN closed supermembrane is based on the light-cone gauge formulation, we apply this formulation to the open supermembrane. We will henceforth use the notation $(\mu) = (+, -, a)$, $X^{\pm} = \frac{1}{\sqrt{2}}(X^1 \pm X^0)$, $a = 2, 3, \ldots, 10$, and $(\xi^i) = (\tau, \sigma^r)$ $(r = 1, 2)$. The light-cone gauge is characterized by the conditions

$$
\begin{aligned}
X^+ &= \tau, \\
\Gamma^+ \theta &= 0.
\end{aligned}
\tag{32}
$$

Following dWHN we further impose the conformal-like gauge conditions

$$
\begin{aligned}
g_{\tau r} &= \partial_r X^- - i\bar{\theta}\Gamma^- \partial_r \theta + \partial_\tau X^a \partial_r X^a = 0, \\
g_{\tau\tau} &= 2(\partial_\tau X^- - i\bar{\theta}\Gamma^- \partial_\tau \theta) + \partial_\tau X^a \partial_\tau X^a \\
&= -\frac{1}{(P_0^+ \sqrt{w})^2} \det(g_{rs}) = -\frac{1}{2(P_0)^2}(\{X^a, X^b\})^2,
\end{aligned}
\tag{33}
$$

where $\sqrt{w}(\sigma)$ is some fixed scalar density on the constant-τ surface $\Sigma^{(2)}$ which is normalized as $\int_{\Sigma^{(2)}} d^2\sigma \sqrt{w(\sigma)} = 1$, P_0^+ is a nonzero constant, and

$$
\{A, B\} = \frac{\epsilon^{rs}}{\sqrt{w}} \partial_r A \partial_s B \tag{34}
$$

stands for the Lie bracket which generates area preserving diffeomorphisms (APD) on $\Sigma^{(2)}$. Substituting these gauge-fixing conditions into the action (18), we find

$$
S = \int d\tau \int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} \left[\frac{1}{2} P_0^+ (\partial_\tau X^a)^2 - i P_0^+ \bar{\theta}\Gamma^- \partial_\tau \theta - \frac{1}{4P_0^+}(\{X^a, X^b\})^2 + i\bar{\theta}\Gamma^- \Gamma^a \{X^a, \theta\} \right]. \tag{35}
$$

By using the nine dimensional spinor notation (see Appendix A.2), it is rewritten as

$$S = \int d\tau \int_{\Sigma^2} d^2\sigma \sqrt{w} \left[\frac{P_0^+}{2} (\partial_\tau X^a)^2 + \frac{i}{2} \theta^T \partial_\tau \theta - \frac{1}{4P_0^+} (\{X^a, X^b\})^2 + \frac{i}{2P_0^+} \theta^T \gamma^a \{X^a, \theta\} \right], \tag{36}$$

where $\theta = (\theta^\alpha)^T$ $(\alpha = 1, 2, \ldots .16)$ is now regarded as a real SO(9) spinor.[6]

From the compatibility between the gauge-fixing conditions (32)(33) and the boundary conditions obtained in the previous section:

$$\begin{aligned} \delta X^{\bar{a}} &= (1 - \Gamma_{(5)})\theta = 0, \\ \partial_1 X^{\underline{\mu}} &= (1 + \Gamma_{(5)})\partial_1 \theta = 0 \quad \text{on } \partial\Sigma, \end{aligned} \tag{37}$$

we see that $X^\pm$ are always parallel to the five-brane world volume. Thus we introduce the notation $(\underline{\mu}) = (+, -, \underline{a})$ with $\underline{a} = 2, 3, 4, 5$.

In order to go further we introduce a metric $w_{rs}(\sigma)$ on $\Sigma^{(2)}$ such that

$$w(\sigma) = \det(w_{rs}(\sigma)), \quad w_{12}|_{\partial\Sigma^{(2)}} = 0, \quad \partial_r(\sqrt{w} w^{r1})|_{\partial\Sigma^{(2)}} = 0. \tag{38}$$

We can then perform the mode expansion

$$\begin{aligned} X^{\bar{a}}(\sigma) &= \sum_A Y_A^{(D)}(\sigma) X^{\bar{a}A}, \quad \theta^{(+)}(\sigma) = \sum_A Y_A^{(D)}(\sigma) \theta^{(+)A}, \\ X^{\underline{a}}(\sigma) &= \sum_A Y_A^{(N)}(\sigma) X^{\underline{a}A}, \quad \theta^{(-)}(\sigma) = \sum_A Y_A^{(N)}(\sigma) \theta^{(-)A}, \end{aligned} \tag{39}$$

where we have used the notation $\theta^{(\pm)} \equiv \frac{1 \pm \gamma_{(4)}}{2} \theta$ with $\gamma_{(4)} = \gamma^{2345}$. We take $Y_A^{(D)}(\sigma)$ and $Y_A^{(N)}(\sigma)$ to be eigenfunctions of the Laplacian

$$\Delta Y_A^{(D,N)} \equiv \frac{1}{\sqrt{w}} \partial_r \left(\sqrt{w} w^{rs} \partial_s Y_A^{(D,N)} \right) = -\omega_A^{(D,N)} Y_A^{(D,N)}$$

which are subject to the Dirichlet and Neumann boundary conditions, respectively. They can be chosen to satisfy the orthonormality

$$\begin{aligned} \int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} Y_A^{(D)} (Y_B^{(D)})^* &= \delta_A^B, \\ \int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} Y_A^{(N)} (Y_B^{(N)})^* &= \delta_A^B. \end{aligned} \tag{40}$$

In general, however, Dirichlet modes and Neumann modes are not orthogonal to each other. While this guarantees the completeness of $\{Y_A^{(D)}(\sigma)\}$ ($\{Y_A^{(N)}(\sigma)\}$) in the space of functions

[6] Relation between the SO(10,1) Majorana spinor and the SO(9) real spinor is given by

$$\theta|_{SO(10,1)} = \frac{1}{2^{3/4}\sqrt{P_0^+}} \begin{pmatrix} 0 \\ \theta|_{SO(9)} \end{pmatrix}.$$

on $\Sigma^{(2)}$ which satisfy the Dirichlet (Neumann) boundary condition,[7] we need an extra care in order to discuss symmetry and dynamics of the open supermembrane.

From the action (36) we can derive the Dirac brackets. We only pick the nonvanishing ones

$$\begin{aligned}
(X^{\bar{a}}(\sigma), P^{\bar{b}}(\sigma'))_{DB} &= \delta^{\bar{a}\bar{b}}\delta^{(D)}(\sigma,\sigma'), \\
(\theta^{(+)}_\alpha(\sigma), \theta^{(+)}_\beta(\sigma'))_{DB} &= \frac{-i}{\sqrt{w(\sigma)}}\left(\frac{1+\gamma_{(4)}}{2}\right)_{\alpha\beta}\delta^{(D)}(\sigma,\sigma'), \\
(X^{\underline{a}}(\sigma), P^{\underline{b}}(\sigma'))_{DB} &= \delta^{\underline{ab}}\delta^{(N)}(\sigma,\sigma'), \\
(\theta^{(-)}_\alpha(\sigma), \theta^{(-)}_\beta(\sigma'))_{DB} &= \frac{-i}{\sqrt{w(\sigma)}}\left(\frac{1-\gamma_{(4)}}{2}\right)_{\alpha\beta}\delta^{(N)}(\sigma,\sigma'),
\end{aligned} \tag{41}$$

where we have defined $P^a \equiv P^+_0\sqrt{w}\partial_\tau X^a$ and

$$\delta^{(D,N)}(\sigma,\sigma') \equiv \sqrt{w(\sigma)}\sum_A Y_A^{(D,N)}(\sigma)(Y_A^{(D,N)}(\sigma'))^*.$$

Time evolution of the system is described by the Hamiltonian

$$\begin{aligned}
H &= \int_{\Sigma(2)} d^2\sigma \frac{\sqrt{w}}{P^+_0}\left[\frac{(P^a)^2}{2w} + \frac{1}{4}(\{X^a, X^b\})^2 - \frac{i}{2}\theta^T\gamma^a\{X^a,\theta\}\right] \\
&= \frac{(P^{\underline{a}}_0)^2 + \mathcal{M}^2}{2P^+_0},
\end{aligned} \tag{42}$$

where $P^{\underline{a}}_0 = \int_{\Sigma^{(2)}} d^2\sigma P^{\underline{a}}(\sigma)$ is the total momentum along the five-brane world volume. We can also regard this equation as the definition of the invariant squared mass $\mathcal{M}^2$ of the open supermembrane. We note that the first equation of (33) implies, as integrability conditions of X^-,

$$\varphi(\sigma) = -\{\frac{P^a}{\sqrt{w}}, X^a\} - \frac{i}{2}\{\theta^T,\theta\} \approx 0, \tag{43}$$

$$\varphi_\lambda = \int d^2\sigma \phi^{(\lambda)^r}\left(P^a\partial_r X^a + \frac{i}{2}\sqrt{w}\theta^T\partial_r\theta\right) \approx 0\,, \tag{44}$$

where $\{\phi^{(\lambda)}_r\}$ is a basis of $H^1(\Sigma^{(2)};\mathbf{R})$. They are regarded as first class constraints which generates area preserving diffeomorphisms

$$\begin{aligned}
\delta_\zeta X^a &= \{\zeta, X^a\}, \\
\delta_{(\lambda)} X^a &= \phi^{(\lambda)r}\partial_r X^a.
\end{aligned} \tag{45}$$

Due to the consideration in the last of subsec.II B, the APD parameter $\zeta(\sigma)$ is subject to the Dirichlet boundary condition

$$\zeta(\sigma) = 0 \quad \text{on } \partial\Sigma. \tag{46}$$

[7] For example, in the interval parametrized by $\sigma^1 \in [0, 1/2]$, $\cos(2\pi\sigma^1) - \cos(6\pi\sigma^1)$ can be expanded in terms of $\sin(2m\pi\sigma^1)$.

As in the closed supermembrane case[13] the theory we constructed can be regarded as a $(0+1)$-dimensional gauge theory with gauge group being the group of area preserving diffeomorphisms which preserve the boundary conditions. The action of the gauge theory is

$$S = \int dt \int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} \left[\frac{1}{2}(D_t X^a)^2 + \frac{i}{2}\theta^T D_t \theta - \frac{1}{4}(\{X^a, X^b\})^2 + \frac{i}{2}\theta^T \gamma^a \{X^a, \theta\} \right], \tag{47}$$

where we have used the covariant derivative such as $D_t X^a = \partial_t X^a - \{\omega, X^a\}$. The gauge transformation of the APD connection $\omega(t,\sigma)$ is given by

$$\delta_\zeta \omega = D_t \zeta = \partial_t \zeta - \{\omega, \zeta\}, \tag{48}$$

where the gauge parameter ζ depends on time in general. We note that, since the connection is a Lie algebra-valued 1-form, it satisfies the Dirichlet boundary condition

$$\omega(t,\sigma) = 0 \quad \text{on } \partial\Sigma. \tag{49}$$

We see that the Hamiltonian (42) and the constraint (43) coincide with those derived from the action (47) *in the $\omega = 0$ gauge*.[8] The residual gauge symmetry is the time-independent APD transformations (45).

As we will see in sec.IV this theory has dynamical supersymmetry generated by the supercharge $Q^+_{(+)}$ which has eight components. Our theory is therefore regarded as a (0+1)-dimensional $N = 8$ supersymmetric gauge theory.

3.2 SO(N) regularization

In ref.[13], de Wit, Hoppe and Nicolai have shown that the closed supermembrane theory is well approximated by U(N) supersymmetric gauge theory in (0+1)-dimension. In this subsection we show that the open supermembrane theory is well approximated by SO(N) supersymmetric gauge theory in (0+1)-dimension. While we only deal with a cylindrical membrane, our analysis can be extended to a general topology if we exploit the mirror image prescription.

First we examine how the matter contents are approximated by SO(N). We introduce the coordinates $\sigma^1 \in [0, \frac{1}{2}]$ and $\sigma^2 (\sim \sigma^2 + 1)$ which respectively parametrize the I- and the S^1-directions of the cylinder $I \times S^1$. It should be kept in mind that we use the following fiducial metric

$$(w_{rs}) = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}.$$

Because we are dealing with the case in which the open supermembrane ends on one five-brane (or on two parallel ones), we have two kinds of mode functions

$$\begin{aligned} \text{Dirichlet (DD sector)} &: \quad Y_A^{(D)}(\sigma^1 = 0, \sigma^2) = Y_A^{(D)}(\sigma^1 = 1/2, \sigma^2) = 0, \\ \text{Neumann (NN sector)} &: \quad \partial_1 Y_A^{(N)}(\sigma^1 = 0, \sigma^2) = \partial_1 Y_A^{(N)}(\sigma^1 = 1/2, \sigma^2) = 0. \end{aligned} \tag{50}$$

[8] Actually the action (36) is equivalent to (47) in the $\omega = 0$ gauge if we realize the time rescaling $t = \tau / P_0^+$.

The Dirichlet and Neumann modes are given by

$$\begin{aligned}
Y_A^{(D)}(\sigma) &= \sqrt{2} e^{2\pi i A_2 \sigma^2} \sin(2\pi A_1 \sigma^1), \\
Y_A^{(N)}(\sigma) &= \sqrt{2} e^{2\pi i A_2 \sigma^2} \cos(2\pi A_1 \sigma^1) \quad \text{for } A_1 \neq 0, \\
Y_{(0,A_2)}^{(N)}(\sigma) &= e^{2\pi i A_2 \sigma^2}.
\end{aligned} \tag{51}$$

This leads us to find the following important correspondence (see Appendix C): [9]

$$\begin{aligned}
\text{Dirichlet modes} &\overset{N\to\infty}{\longleftarrow} N \times N \text{ antisymmetric matrices}, \\
\text{Neumann modes} &\overset{N\to\infty}{\longleftarrow} N \times N \text{ symmetric matrices}.
\end{aligned} \tag{52}$$

Group structure of the APD gives another support for the SO(N$\to$ ∞) approximation. Namely, the Lie bracket has the following property

$$\begin{aligned}
\{\text{Dirichlet}, \text{Dirichlet}\} &\sim \text{Dirichlet}, \\
\{\text{Dirichlet}, \text{Neumann}\} &\sim \text{Neumann}, \\
\{\text{Neumann}, \text{Neumann}\} &\sim \text{Dirichlet}.
\end{aligned} \tag{53}$$

This coincides with the structure of commutation relations for $N \times N$ matrices. Actually we can see that the large N limit of the commutation relations reproduces the corresponding APD brackets.

However, we need a careful consideration with regard to the matrix regularization of the action, constraints and conserved charges. As is already pointed out in subsec.III A, Dirichlet modes and Neumann ones are not orthogonal w.r.t. the integration $\int_{\Sigma^{(2)}} d^2\sigma$, while antisymmetric matrices and symmetric ones are orthogonal to each other w.r.t. the trace of $N \times N$ matrices. This tells us that we cannot naively replace the integral $\int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} A(\sigma) B(\sigma)$ by the trace $\mathrm{Tr}(AB)$. In the next subsection we will propose a few redefinitions of the "trace" which should be used to define the Lagrangian. However, the following argument shows that we can nevertheless use the original "naive" definition of the trace as long as we use it to approximate the integral in the Lagrangian, smeared constraints, and conserved charges.

By inspecting the action (36), we find that it has the structure

$$\int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} \left[(\text{Dirichlet}) \times (\text{Dirichlet}) + (\text{Neumann}) \times (\text{Neumann}) \right], \tag{54}$$

and that it never contains terms like $\int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} (\text{Dirichlet}) \times (\text{Neumann})$. This is true also for constraints and conserved charges because of the following reasoning. They become generators of some transformations and physically relevant transformations must preserve the boundary conditions. From the Dirac brackets (41) we see that such generators must be of the structure (54). What remains to be shown is that the integration $\int_{\Sigma^{(2)}} d^2\sigma \sqrt{w}$ indeed has properties of the trace for these restricted situations. This is shown in Appendix D.

We can now give the explicit form of the regularized theory. We replace the real functions on $\Sigma^{(2)}$ by $N \times N$ hermitian matrices. The action and the constraint are given by

[9] Similar correspondence relation has been found by Kim and Rey in a slightly different context[17].

replacing $\int_{\Sigma^{(2)}} d^2\sigma\sqrt{w}(A(\sigma)B(\sigma))$ and $\{A,B\}$ in eqs.(47) and (43) with $\mathrm{Tr}(AB)$ and $i[A,B]$, respectively. We find

$$\begin{aligned} S &= \int dt \mathrm{Tr}\left(\frac{1}{2}(D_tX^a)^2 + \frac{i}{2}\theta^T D_t\theta + \frac{1}{4}([X^a,X^b])^2 - \frac{1}{2}\theta^T\gamma^a[X^a,\theta]\right), \\ \varphi &= -i[P^a,X^a] + \frac{1}{2}[\theta^\alpha,\theta^\alpha]_+, \end{aligned} \tag{55}$$

where we have defined the covariant derivative $D_tA = \partial_tA - i[\omega, A]$ with an SO(N) connection $\omega(t)$, and introduced the anticommutator $[A,B]_+ \equiv AB + BA$ of the matrices A and B. $P^a = D_tX^a$ is the momentum conjugate to X^a. Needless to say, the Gauss law constraint φ generates SO(N) gauge transformations. In terms of the SO(N) representation, the matter contents are classified as

$$\begin{array}{lcll} \text{Adjoint} & : & \omega, X^6, X^7, X^8, X^9, X^{10}, & \theta^{(+)}, \\ \text{Symmetric rank-2} & : & X^2, X^3, X^4, X^5, & \theta^{(-)}. \end{array} \tag{56}$$

We note that the fermion $\theta^{(+)}$ (or $\theta^{(-)}$) corresponds to the 4 real canonical pairs and that the two bosonic canonical pairs in the adjoint representation are absorbed into gauge degrees of freedom. Thus we find that, up to a finite number associated with zero-modes, bosonic and fermionic degrees of freedom precisely match with each other and thus supersymmetry is expected to hold in a rigorous sense.

We may interpret the matter content (56) in terms of a (5+1)-dimensional theory. The adjoint matter corresponds to the 6D N=1 vector multiplet and thus it is considered to be obtained from a 6D N=1 super Yang-Mills field. The matter in the symmetric representation is regarded as coming from a 6D N=1 hyper multiplet. Let us next consider the number of the generators of the dynamical supersymmetry. The dWHN closed supermembrane has 16 generators of dynamical supersymmetry, corresponding to N=1 SUSY in 10D.[10] In the open supermembrane case, symmetry generated by a half of them is broken due to the boundary (see sec.IV). Therefore this theory has dynamical supersymmetry generated by 8 supercharges, corresponding to N=1 SUSY in 6D. From these indications it would be plausible to consider the matrix theory(55) to be the dimensional reduction to (0+1)-dimension of the (5+1)-dimensional $SO(N)$ N=1 supersymmetric Yang-Mills theory with a hyper multiplet in the rank-2 symmetric representation. [11]

To conclude this subsection we make a remark. In the cylindrical membrane, there is one more constraint associated with rotation along the S^1-direction. It plays an important role if we compactify the dimensions parallel to the five-brane world volume. It is given by

$$\varphi_2 = \int d^2\sigma\sqrt{w}\left[P_0^+\partial_2X^- + \frac{P^a}{\sqrt{w}}\partial_2X^a + \frac{i}{2}\theta^T\partial_2\theta\right] \approx 0. \tag{57}$$

As in ref.[20] it is in principle possible to consider a matrix version of this constraint. In this paper, however, we will not pursue this issue any more.

[10] This type of supersymmetric gauge quantum mechanics was first discussed in ref.[18]

[11] When we quantize the fermionic zero-modes $\theta_0^{(-)} \sim \mathrm{Tr}\theta^{(-)}$, we have a 6D N=1 tensor multiplet and a hypermultiplet both of which are $SO(N)$ singlet [8]. They are expected to yield the collective coordinates of the five-brane [19].

3.3 Regularization via a non-commutative cylinder

In the last subsection we proposed an SO(N) regularization of the open supermembrane. As we have seen, however, we cannot obtain the complete correspondence between the integration and the trace in this regularization. This is unsatisfactory if one wants to regularize the theory of open supermembranes by means of a "non-commutative cylinder". In this subsection we give a few proposals to modify the definition of the trace to make a consistent correspondence with the integral. The ambiguity comes from the formula,

$$\int_0^{1/2} d\sigma e^{2\pi i m\sigma} = \begin{cases} 1/2 & m=0 \\ i\frac{1-(-1)^m}{2\pi m} & m\neq 0. \end{cases} \tag{58}$$

Obviously the usual definition of the trace gives $TrU^m = N\delta_{m0}$ (mod N) and it cannot give the second term of (58).

To begin with we show that the modified definition of the trace Tr' can not satisfy the fundamental property of the trace,

$$\mathrm{Tr}'(AB) = \mathrm{Tr}'(BA). \tag{59}$$

This is because the commutation relation $VUV^{-1} = \omega U$ ($\omega = e^{2\pi i/N}$) and $\mathrm{Tr}'U \neq 0$ are not consistent with (59). In this sense, the problem is similar to the definition of p and q with $[q,p] = I_N$ whose realization is impossible in the finite N.

This observation leads us to give a modified definition of the trace as follows. Let us consider the case when N is even, namely $N \equiv 2M$. The modified definition of the trace may be given as

$$\mathrm{Tr}'A = \mathrm{Tr}\mathcal{P}A, \quad \mathcal{P} = \begin{pmatrix} I_M & 0_M \\ 0_M & 0_M \end{pmatrix}. \tag{60}$$

It gives

$$\frac{1}{N}\mathrm{Tr}'U^m V^n = \begin{cases} \frac{1}{2}\delta_{n0} & m=0 \mod N \\ \delta_{n0}\frac{1-(-1)^m}{N(1-\omega^m)} & m\neq 0 \mod N \end{cases}. \tag{61}$$

This is obviously consistent with (58) in the large N limit. Although the relation (59) is violated in the finite N, the anomalous components appear only at the "boundary" of the $M \times M$ blocks. Such terms are supposed to disappear in the large N limit.

Another possible redefinition is to keep the definition of the trace but to redefine the generator which corresponds to $e^{2\pi i\sigma_1/N}$. Instead of using U, we introduce the square root,

$$U' = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \omega' & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots & (\omega')^{N-1} \end{pmatrix} \qquad \omega' = e^{\pi i/N}. \tag{62}$$

By easy manipulation, one can prove the consistency with (58) and (59). However, the commutation relation $VU' = \omega' U' V$ is violated. As in the previous redefinition, the anomalous components appear at the boundary of the matrix and will disappear in the large N limit.

The relation with the last subsection is clearer in the first redefinition. What we have proved in sec.III B is that, in the definition of the lagrangian etc., one may effectively replace Tr' with the ordinary trace since there are no integrand of the (Dirichlet)$\times$(Neumann) type.

4 11D SUSY algebra in the light-cone gauge

In this section we investigate the supersymmetry algebra of the model constructed in subsec.III A. Extension to the matrix version is straightforward.

In the case of a closed supermembrane, there are two kinds of supercharges

$$\begin{aligned}
Q^- &= \sqrt{P_0^+}\int d^2\sigma\sqrt{w}\theta, \\
Q^+ &= \frac{1}{\sqrt{P_0^+}}\int d^2\sigma\left(P^a\gamma_a + \frac{\sqrt{w}}{2}\{X^a, X^b\}\gamma_{ab}\right)\theta.
\end{aligned} \tag{63}$$

They respectively generate kinematical SUSY transformations

$$\delta_- X^a = 0, \quad \delta_-\theta = \epsilon', \tag{64}$$

and dynamical SUSY transformations

$$\delta_+ X^a = \epsilon\gamma^a\theta, \quad \delta_+\theta = +i\left(\frac{P^a}{\sqrt{w}}\gamma_a - \frac{1}{2}\{X^a, X^b\}\gamma_{ab}\right)\epsilon, \tag{65}$$

where ϵ' and ϵ are constant real spinors of SO(9).

In the case of an open supermembrane, however, physically relevant SUSY transformations have to preserve boundary conditions. We are thus left with the following generators

$$\begin{aligned}
Q^-_{(-)} &\equiv \frac{1-\gamma_{(4)}}{2}Q^-, \\
Q^+_{(+)} &\equiv \frac{1+\gamma_{(4)}}{2}Q^+.
\end{aligned} \tag{66}$$

By using the Dirac brackets (41) we have confirmed that these generators are well-defined and that the action (36) is invariant under the SUSY transformations generated by them even if the boundary terms are taken into account. This is not surprising because these generators are of the form (54).

The algebra formed by the generators (66) of unbroken SUSY is found to be

$$\begin{aligned}
i(Q^-_{(-)\alpha}, Q^-_{(-)\beta})_{DB} &= (\mathcal{P}^{(-)})_{\alpha\beta}P_0^+, \\
i(Q^-_{(-)\alpha}, Q^+_{(+)\beta})_{DB} &= (\mathcal{P}^{(-)}\gamma_{\underline{a}})_{\alpha\beta}P_0^{\underline{a}} + (\mathcal{P}^{(-)}\gamma_{\bar{a}\underline{b}})_{\alpha\beta}Z^{\bar{a}\underline{b}}, \\
i(Q^+_{(+)\alpha}, Q^+_{(+)\beta})_{DB} &= 2H(\mathcal{P}^{(+)})_{\alpha\beta} + 2(\mathcal{P}^{+}\gamma_{\bar{a}})_{\alpha\beta}Z^{\bar{a}},
\end{aligned} \tag{67}$$

where we have introduced the projection operators $\mathcal{P}^{(\pm)} \equiv \frac{I_{16}\pm\gamma_{(4)}}{2}$, and membrane charges are given by

$$\begin{aligned}
Z^{ab} &= -\int d^2\sigma\sqrt{w}\{X^a, X^b\}, \\
Z^a &= \frac{1}{P_0^+}\int d^2\sigma\sqrt{w}\left(\{X^a, X^b\}\frac{P^b}{\sqrt{w}} + \frac{i}{2}\theta^T\{X^a, \theta\}\right) \\
&= -\int d^2\sigma\sqrt{w}\{X^a, X^-\}.
\end{aligned} \tag{68}$$

In the case of the closed supermembrane we can use Q^+ and Q^- to construct the 11 dimensional SUSY generator

$$Q \equiv \begin{pmatrix} \sqrt{2}Q^- \\ Q^+ \end{pmatrix}. \tag{69}$$

The generators of unbroken SUSY (66) in the open membrane case are then interpreted as those resulting from the projection

$$\tilde{Q} \equiv \begin{pmatrix} \sqrt{2}Q^-_{(-)} \\ Q^+_{(+)} \end{pmatrix} = \frac{1-\Gamma_{(5)}}{2}Q. \tag{70}$$

By virtue of this fact the SUSY algebra (67) is rewritten in a 11 dimensional form

$$i(\tilde{Q},\tilde{Q}^T)_{DB} = \begin{pmatrix} 2P_0^+\mathcal{P}^{(-)} & \sqrt{2}\mathcal{P}^{(-)}(\not{P}+\not{Z}_{(2)}) \\ \sqrt{2}\mathcal{P}^{(+)}(\not{P}-\not{Z}_{(2)}) & 2\mathcal{P}^{(+)}(H\cdot I_{16}+\not{Z}_{(1)}) \end{pmatrix} \tag{71}$$

$$= \begin{pmatrix} \sqrt{2}P_0^+\mathcal{P}^{(-)} & 0 \\ \mathcal{P}^{(+)}(\not{P}-\not{Z}_{(2)}) & \mathcal{P}^{(+)} \end{pmatrix} \begin{pmatrix} \frac{1}{P_0^+}\mathcal{P}^{(-)} & 0 \\ 0 & \frac{1}{P_0^+}\mathbf{m} \end{pmatrix} \begin{pmatrix} \sqrt{2}P_0^+\mathcal{P}^{(-)} & \mathcal{P}^{(-)}(\not{P}+\not{Z}_{(2)}) \\ 0 & \mathcal{P}^{(+)} \end{pmatrix},$$

where we have defined the matrices $\not{P} = P_0^{\underline{a}}\gamma_{\underline{a}}$, $\not{Z}_{(2)} = Z^{\bar{a}\underline{b}}\gamma_{\bar{a}\underline{b}}$, $\not{Z}_{(1)} = Z^{\bar{a}}\gamma_{\bar{a}}$, and

$$\begin{aligned} \mathbf{m} &\equiv \mathcal{P}^{(+)}\left[2P_0^+(H\cdot I_{16}+\not{Z}_{(1)}) - (\not{P}-\not{Z}_{(2)})(\not{P}+\not{Z}_{(2)})\right] \\ &= \mathcal{P}^{(+)}\left[(\mathcal{M}^2 - Z^{\bar{a}\underline{b}}Z^{\bar{a}\underline{b}})I_{16} + 2(Z^{\bar{a}}P_0^+ + Z^{\bar{a}\underline{c}}P_0^{\underline{c}})\gamma_{\bar{a}} - Z^{\bar{a}\underline{b}}Z^{\bar{c}\underline{d}}\gamma_{\bar{a}\bar{c}\underline{b}\underline{d}}\right]. \end{aligned} \tag{72}$$

Here we make a comment on the five-brane charges. In the present case, 11D SUSY generated by Q is broken to that generated by $\tilde{Q} = \frac{1-\Gamma_{(5)}}{2}Q$, as is expected from the result in subsec.II B. This pattern of breakdown of supersymmetry coincides with that in the presence of five-branes with charges

$$P^0 = Z^{12345}(=\infty).$$

In this sense we can say that our theory of open supermembrane effectively incorporates the longitudinal five-brane charge Z^{-2345} $(= Z^{+2345})$. We should remark that we cannot incorporate the transverse five-brane charge because the gauge-fixing conditions (32)(33) imply that $X^\pm$ be subject to the Neumann boundary condition. This partially agrees with the statements in ref.[21]. There is, however, an essential difference. Namely we cannot give an explicit expression of the five-brane charge from the bulk such as $Z^{-abcd} \sim \mathrm{Tr}(X^{[a}X^bX^cX^{d]})$ which was discussed in [21]. This is because the corresponding expression in the membrane theory

$$\int d^2\sigma\sqrt{w}\{X^{[a},X^b\}\{X^c,X^{d]}\},$$

always vanishes due to the identity $\epsilon^{r[s}\epsilon^{tu]} = 0$. We should rather identify the five brane charge as coming from the topological defect at the boundary.

4.1 BPS configurations

Now that we have the 11D SUSY algebra, let us explore BPS conditions. From the analysis of the SUSY algebra we see that the nontrivial BPS configurations with nonvanishing membrane

charges should stretch both in the directions parallel and perpendicular to the five-brane(s). It is therefore sufficient to consider the situation in which there are two parallel five-branes and an open membrane which stretches between them. Without loss of generality we can set the boundary conditions

$$X^{\bar{a}}|_{\sigma^1=0} = 0, \quad X^{\bar{a}}|_{\sigma^1=1/2} = b\delta^{\bar{a}}_{10}, \quad \partial_1 X^{\underline{a}}|_{\sigma^1=0} = \partial_1 X^{\underline{a}}|_{\sigma^1=1/2} = 0, \tag{73}$$

and so on. In order to obtain nonvanishing membrane charges we further have to consider either the case in which: (i) the membrane stretches infinitely along the five-branes; or (ii) the membrane wrapps around a 1-dimensional cycle which is parallel to the five-branes. We analyze the case (ii) in order to avoid the divergence of the membrane charges. Because we are dealing with flat five-branes we toroidally compactify the directions parallel to the five-branes

$$X^{\underline{a}} \sim X^{\underline{a}} + 2\pi R^{\underline{a}}, \quad X^- \sim X^- + 2\pi R. \tag{74}$$

This also serves as a regularization of the case (i).

Thus we have $P_0^+ = \frac{m}{R}$ and

$$\begin{aligned} X^{\underline{a}} &= \frac{Rm^{\underline{a}}}{R^{\underline{a}}m}\tau + 2\pi R^{\underline{a}} n^{\underline{a}} \sigma^2 + X_0^{\underline{a}} + \hat{X}^{\underline{a}}(\tau,\sigma), \\ X^{\bar{a}} &= 2b\delta^{\bar{a}}_{10}\sigma^1 + \hat{X}^{\bar{a}}(\tau,\sigma), \\ X^- &= -\frac{R}{m}H\tau + 2\pi Rn\sigma^2 + \hat{X}^-(\tau,\sigma), \\ \theta^{(-)} &= \theta_0^{(-)} + \hat{\theta}^{(-)}(\tau,\sigma), \quad \theta^{(+)} = \hat{\theta}^{(+)}(\tau,\sigma), \end{aligned} \tag{75}$$

where the hat stands for the oscillating part. In this notation, constraints are rewritten as

$$\begin{aligned} 0 &\approx \varphi(\sigma) = \nabla^a\left(\frac{\hat{P}^a}{\sqrt{w}}\right) + \{\hat{X}^a, \frac{\hat{P}^a}{\sqrt{w}}\} - \frac{i}{2}\{\hat{\theta}^T, \hat{\theta}\}, \\ 0 &\approx \varphi_2 = 2\pi(nm + n^{\underline{a}}m^{\underline{a}}) + \int d^2\sigma(\hat{P}^a\partial_2\hat{X}^a + \frac{i}{2}\sqrt{w}\hat{\theta}^T\partial_2\hat{\theta}), \end{aligned} \tag{76}$$

where $\nabla^{\underline{a}} \equiv -\pi R^{\underline{a}} n^{\underline{a}}\partial_1$ and $\nabla^{\bar{a}} \equiv b\delta^{\bar{a}}_{10}\partial_2$. We can also calculate membrane charges

$$\begin{aligned} Z^{\bar{a}\bar{b}} &= Z^{\underline{a}\underline{b}} = Z^{\underline{a}} = 0, \\ Z^{\bar{a}\underline{b}} &= -(2\pi b R^{\underline{b}} n^{\underline{b}})\delta^{\bar{a}}_{10}, \\ Z^{\bar{a}} &= -(2\pi bRn)\delta^{\bar{a}}_{10}. \end{aligned} \tag{77}$$

Now we can identify the BPS configurations. Let us start with the configuration which preserves 1/4 SUSY. Such a BPS configuration should make the matrix $\mathbf{m}$ (eq.(72)) vanish. Because the last term in eq.(72) always vanishes in the cylindrical membrane, we have the following BPS conditions

$$\begin{aligned} 0 &= \mathcal{M}^2 - Z^{\bar{a}\underline{b}}Z^{\bar{a}\underline{b}} \\ &= \int d^2\sigma\sqrt{w}\left[(\hat{P}^a/\sqrt{w})^2 + \frac{1}{2}(\{X^a, X^b\} + Z^{ab})^2 - i\hat{\theta}^T\gamma^a\{X^a, \hat{\theta}\}\right], \\ 0 &= Z^{\bar{a}}P_0^+ + Z^{\bar{a}\underline{c}}P_0^{\underline{c}} = -2\pi b\delta^{\bar{a}}_{10}(nm + n^{\underline{c}}m^{\underline{c}}) \\ &\approx b\delta^{\bar{a}}_{10}\int d^2\sigma(\hat{P}^a\partial_2\hat{X}^a + \frac{i}{2}\sqrt{w}\hat{\theta}^T\partial_2\hat{\theta}). \end{aligned} \tag{78}$$

As a general solution to these conditions, we find the BPS configuration with 1/4 SUSY:

$$\begin{aligned}
X^{\underline{a}} &= \frac{Rm^{\underline{a}}}{R^{\underline{a}}m}\tau + 2\pi R^{\underline{a}} n^{\underline{a}}\sigma^2, \quad X^{\bar{a}} = 2b\delta^{\bar{a}}_{10}\sigma^1, \\
X^- &= -\frac{R}{m}H\tau + 2\pi Rn\sigma^2 \quad (\text{with } mn + n^{\underline{a}}m^{\underline{a}} = 0), \\
\theta^{(-)} &= \theta^{(-)}_0, \quad \theta^{(+)} = 0.
\end{aligned} \tag{79}$$

It represents a (2+1)-dimensional hyperplane-like membrane which stretches between the two five-branes. It should be closely related to the "intersecting-brane" configurations[22]. A matrix version of this configuration corresponds to the "open membrane in M(atrix) theory"[23].

Next we consider the configuration with 1/8 SUSY. In such a configuration the rank of the 16×16 matrix $\mathbf{m}$ becomes 4. The BPS bound is given by

$$\mathcal{M}^2 - Z^{\bar{a}\underline{b}}Z^{\bar{a}\underline{b}} \mp 2(Z^{10}P^+_0 + Z^{10\underline{c}}P^{\underline{c}}_0) = 0. \tag{80}$$

The analysis is almost parallel to that of BPS states with 1/4 SUSY for the closed supermembrane[20]. We find, in the case $\hat{\theta}^{(\pm)} = 0$, the following BPS conditions

$$\begin{aligned}
\hat{P}^{10} &= 0, \\
\frac{\hat{P}^i}{\sqrt{w}} &= \pm(\{X^{10}, X^i\} + Z^{10i}), \\
0 &= \{X^i, X^j\} + Z^{ij},
\end{aligned} \tag{81}$$

where $i, j = 2, 3, \ldots, 9$. The generator of unbroken SUSY is given by

$$\tilde{Q}^{(\mp)} \equiv \mathcal{P}^{(+)}\frac{1 \mp \gamma_{10}}{2}\int d^2\sigma \left[\hat{P}^a\gamma_a + \frac{\sqrt{w}}{2}(\{X^a, X^b\} + Z^{ab})\gamma_{ab}\right]\hat{\theta}. \tag{82}$$

We can see that eq.(81) is equivalent to the condition $(\tilde{Q}^{(\mp)}, \hat{\theta})_{DB} = 0$.[12]

As in ref.[20] we can provide an example of the configurations with 1/8 SUSY with nonvanishing $\hat{\theta}$.[13] We consider the following "dimensional reduction" of the membrane world-volume

$$\begin{aligned}
X^{\bar{a}} &= 2b\delta^{\bar{a}}_{10}\sigma^1, \\
X^{\underline{a}} &= \frac{Rm^{\underline{a}}}{R^{\underline{a}}m}\tau + 2\pi R^{\underline{a}}n^{\underline{a}}\sigma^2 + \hat{X}^{\underline{a}}(\tau, \sigma^2),
\end{aligned}$$

[12] One might claim that the latter only imposes $\{X^{\underline{a}}, X^{\underline{b}}\} = \frac{1}{2}\epsilon^{\underline{abcd}}\{X^{\underline{c}}, X^{\underline{d}}\}$ instead of $\{X^{\underline{a}}, X^{\underline{b}}\} = 0$ when the target space is T^d with $d \geq 4$. However, these two equations turn out to be equivalent by virtue of the identity

$$\int d^2\sigma\sqrt{w}\epsilon^{\underline{abcd}}\{X^{\underline{a}}, X^{\underline{b}}\}\{X^{\underline{c}}, X^{\underline{d}}\} = 0.$$

[13] As a matter of fact we can show that this example yields an almost general solution to the BPS conditions (81). For a detailed analysis, we refer the reader to Appendix E.

$$
\begin{aligned}
X^- &= -\frac{R}{m}H\tau + 2\pi Rn\sigma^2 + \hat{X}^-(\tau,\sigma^2), \\
\theta^{(+)} &= 0, \\
\theta^{(-)} &= \theta_0^{(-)} + \hat{\theta}^{(-)}(\tau,\sigma^2).
\end{aligned} \tag{83}
$$

The constraints and the BPS conditions are reduced to the following form:

$$
\begin{aligned}
0 &\approx \varphi_2 = 2\pi(nm + n^{\underline{a}}m^{\underline{a}}) + \oint d\sigma^2 \left(\frac{\hat{P}^{\underline{a}}}{\sqrt{w}}\partial_2\hat{X}^{\underline{a}} + \frac{i}{2}\hat{\theta}^{(-)T}\partial_2\hat{\theta}^{(-)} \right), \\
\partial_\tau\hat{X}^{\underline{a}} &= \pm\frac{Rb}{m}\partial_2\hat{X}^{\underline{a}}, \\
\hat{\theta}^{(-)} &= \mp\gamma_{10}\hat{\theta}^{(-)}.
\end{aligned} \tag{84}
$$

This configuration represents an interval times a closed string which is composed only of the right-(left-)moving modes. In the limit $b \to 0$ it yields a tensionless string which is static on the five-brane world volume.

5 Discussions

In this paper we have investigated open supermembranes in the 11D rigid superspace. We have seen that kappa-symmetry and invariance under a fraction of 11D SUSY specify the Dirichlet boundary conditions. The conditions for the fermion seem to enforce the "self-duality" of the three-form field strength on the five-brane world volume. In retrospect, kappa-symmetry of the closed supermembrane in a curved background required the background be a solution of 11D supergravity[9]. In this sense kappa-symmetry of the supermembrane theory in the covariant formalism plays a role similar to that of the conformal invariance in superstring theory. This reasoning leads us to the conjecture that kappa-symmetry of open supermembranes in a curved background yields the field equations for the (collective modes of) M-theory five-branes. It would be interesting to pursue this possibility. [14]

We have also shown that the light-cone gauge formulation is regularized by a (0+1)-dimensional SO(N) supersymmetric gauge theory. It is known that the matrix regularization of the closed supermembrane is closely related to the matrix formulation of M-theory [24]. Because our SO(N$\to\infty$) theory describes an open supermembrane and a five-brane which are also essential to the description of M-theory, it is conceivable that the true M(atrix) theory incorporates naturally the SO(N$\to\infty$) theory in a certain sense.

We should remark that our analysis is classical and thus we do not consider the effect of anomalies. Because the boundary of the open membrane is two dimensional and because the fields on the five-brane world volume is chiral, anomalies are expected to arise[3][10]. It is an important task to examine what modification is required if anomalies are taken into account.[15] [16] In particular, it would be of interest to see whether the $p=1$ case survives

[14] Quite recently Chu and Sezgin demonstrated that our conjecture is indeed true [25].

[15] A related topic is the M(atrix) theory compactified on $T^5/\mathbf{Z}_2$ which is described by a USp(2N) supersymmetric gauge theory[26]. It might be worthwhile constructing an anomaly-free theory by combining this USp(2N) model with our SO(N) model.

[16] Recently Brax and Mourad have analyzed in detail the issue of anomalies in the theory of open super-

through anomalies.

While we have concentrated on the $p=5$ case, our result may be extended to the $p=9$ case. This case is interesting because it is related to the Matrix theory of heterotic strings [28][17]. Finally we will briefly discuss their relation. Boundary conditions for the open supermembrane which ends on the 9-brane are given by

$$\begin{aligned} \text{Dirichlet} &: \omega, \quad X^{10}, \quad \frac{1-\Gamma_{(9)}}{2}\theta; \\ \text{Neumann} &: X^{\underline{a}} \quad (\underline{a}=2,3,\ldots,9), \quad \frac{1+\Gamma_{(9)}}{2}\theta, \end{aligned} \tag{85}$$

After performing the matrix regularization this agrees with the matter contents of [28][17] because Dirichlet and Neumann modes are respectively approximated by $N \times N$ antisymmetric and symmetric matrices. This gives a support for the idea of the heterotic matrix theory from a different point of view. It would therefore be intriguing to further investigate this model from the membrane side.

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A Convention and useful formulae

A.1 SO(10,1) Clifford algebra

Eleven dimensional gamma matrices Γ^μ $(\mu=0,1,\ldots,10)$ satisfy the SO(10,1) Clifford algebra

$$\Gamma^\mu\Gamma^\nu = \eta^{\mu\nu} I_{32} + \Gamma^{\mu\nu}, \tag{86}$$

where $(\eta^{\mu\nu}) = \mathrm{diag}(-,+,\ldots,+)$ denotes the eleven dimensional Minkowski metric and we use the notation

$$\Gamma^{\mu_1\cdots\mu_n} \equiv \Gamma^{[\mu_1}\cdots\Gamma^{\mu_n]}.$$

We should remark that Γ^{10} is identified with the ten dimensional chiral matrix $\Gamma_{11} = \Gamma_{01\cdots 9}$. If we define the charge conjugation matrix $\mathcal{C}$ by

$$\mathcal{C}^{-1}(\Gamma^\mu)^T\mathcal{C} = -\Gamma^\mu, \tag{87}$$

the gamma matrices have the following properties

$$\begin{aligned} &\mathcal{C}^{-1}(\Gamma^{\mu_1\cdots\mu_n})^T\mathcal{C} = (-)^{\frac{n(n+1)}{2}}\Gamma^{\mu_1\cdots\mu_n}, \\ &(\mathcal{C}\Gamma_{\mu\nu})_{(\alpha\beta}(\mathcal{C}\Gamma^\nu)_{\gamma\delta)} = 0. \end{aligned} \tag{88}$$

From the first equation it follows

$$\mathcal{C}^{-1}(\Gamma_{(p)})^T\mathcal{C} = (-)^p(\Gamma_{(p)})^{-1}, \tag{89}$$

membranes which end on the five-branes[27].

where $\Gamma_{(p)} \equiv \Gamma_{01\cdots p}$.

In practical calculation it is frequently convenient to use the Majorana representation in which the spinor is real and

$$\Gamma^0 = \mathcal{C} = \begin{pmatrix} 0 & I_{16} \\ -I_{16} & 0 \end{pmatrix}, \quad \Gamma^1 = \begin{pmatrix} 0 & -I_{16} \\ -I_{16} & 0 \end{pmatrix}, \quad \Gamma^a = \begin{pmatrix} \gamma^a & 0 \\ 0 & -\gamma^a \end{pmatrix}, \tag{90}$$

where γ^a ($a = 2, 3, \ldots, 10$) are SO(9) gamma matrices in the real representation in which γ^a are real and symmetric.

A.2 SO(5,1)⊂SO(10,1) Clifford algebra

Among the eleven dimensional gamma matrices, $\Gamma^{\underline{\mu}}$ ($\underline{\mu} = 0, 1, \ldots, 5$) form an SO(5,1) Clifford algebra. The 6D chirality is defined by the matrix

$$\Gamma_{(5)} = \Gamma_{01\cdots 5} = \frac{\epsilon^{\underline{\mu\nu\rho\sigma\kappa\lambda}}}{6!}\Gamma_{\underline{\mu\nu\rho\sigma\kappa\lambda}}, \tag{91}$$

which satisfies

$$(\Gamma_{(5)})^2 = I_{32}, \quad \Gamma_{(5)}\Gamma^{\underline{\mu}} + \Gamma^{\underline{\mu}}\Gamma_{(5)} = 0. \tag{92}$$

From the definition (91) we find the 'duality' relations which are useful in the analysis of open supermembranes

$$\begin{aligned} \Gamma_{\underline{\mu\nu\rho\sigma\kappa\lambda}} &= -\epsilon_{\underline{\mu\nu\rho\sigma\kappa\lambda}}\Gamma_{(5)}, \\ \Gamma_{\underline{\mu\nu\rho\sigma\kappa}} &= -\epsilon_{\underline{\mu\nu\rho\sigma\kappa\lambda}}\Gamma_{(5)}\Gamma^{\underline{\lambda}}, \\ \Gamma_{\underline{\mu\nu\rho\sigma}} &= \frac{1}{2}\epsilon_{\underline{\mu\nu\rho\sigma\kappa\lambda}}\Gamma_{(5)}\Gamma^{\underline{\kappa\lambda}}, \\ \Gamma_{\underline{\mu\nu\rho}} &= \frac{1}{3!}\epsilon_{\underline{\mu\nu\rho\sigma\kappa\lambda}}\Gamma_{(5)}\Gamma^{\underline{\sigma\kappa\lambda}}. \end{aligned} \tag{93}$$

A.3 Self-dual three-form $h_{\underline{\mu\nu\rho}}$

In the analysis of open supermembranes we frequently deal with the self-dual three-form on the five-brane world volume

$$h_{\underline{\mu\nu\rho}} = \frac{1}{3!}\epsilon_{\underline{\mu\nu\rho\sigma\kappa\lambda}}h^{\underline{\sigma\kappa\lambda}}. \tag{94}$$

If we define the tensor $k_{\underline{\nu}}^{\underline{\mu}} \equiv h^{\underline{\mu\rho\sigma}}h_{\underline{\nu\rho\sigma}}$, we find the following useful identities

$$\begin{aligned} k_{\underline{\mu}}^{\underline{\mu}} &= h^{\underline{\mu\nu\rho}}h_{\underline{\mu\nu\rho}} = 0, \\ h^{\underline{\mu\nu\kappa}}h_{\underline{\rho\sigma\kappa}} &= \delta_{[\underline{\rho}}^{[\underline{\mu}}k_{\underline{\sigma}]}^{\underline{\nu}]}, \\ k_{\underline{\sigma}}^{\underline{\mu}}h^{\underline{\sigma\nu\rho}} &= k_{\underline{\sigma}}^{\underline{\nu}}h^{\underline{\mu\sigma\rho}}, \\ h^{\underline{\mu}[\underline{\nu}}{}_{\underline{\kappa}}h^{\underline{\rho\sigma}]\underline{\kappa}} &= 0, \\ k_{\underline{\mu}}^{\underline{\rho}}k_{\underline{\rho}}^{\underline{\nu}} &= \frac{1}{6}\delta_{\underline{\mu}}^{\underline{\nu}}(k_{\underline{\sigma}}^{\underline{\rho}}k_{\underline{\rho}}^{\underline{\sigma}}), \\ k_{\underline{\mu}}^{\underline{\sigma}}k_{\underline{\nu}}^{\underline{\kappa}}h_{\underline{\sigma\kappa\rho}} &= \frac{1}{6}(k_{\underline{\kappa}}^{\underline{\lambda}}k_{\underline{\lambda}}^{\underline{\kappa}})h_{\underline{\mu\nu\rho}}. \end{aligned} \tag{95}$$

B Derivation of eq.(28)

In this appendix we reproduce eq.(24) from eq.(27). We first note that (27) is rewritten as

$$\theta' \equiv \exp(\frac{1}{3!}h_{\underline{\mu\nu\rho}}\Gamma^{\underline{\mu\nu\rho}})\theta = \Gamma_{(5)}\theta'. \tag{96}$$

Thus we find

$$\begin{aligned} 0 &= \bar{\theta}'\Gamma_{\underline{\mu\nu}}\delta\theta' \\ &= \bar{\theta}\exp\left(\frac{1}{3!}h_{\underline{\rho\sigma\kappa}}\Gamma^{\underline{\rho\sigma\kappa}}\right)\Gamma_{\underline{\mu\nu}}\exp\left(\frac{1}{3!}h_{\underline{\rho\sigma\kappa}}\Gamma^{\underline{\rho\sigma\kappa}}\right)\delta\theta \\ &= \bar{\theta}\Gamma_{\underline{\mu\nu}}\delta\theta - 2h_{\underline{\mu\nu\rho}}\bar{\theta}\Gamma^{\underline{\rho}}(1+\Gamma_{(5)})\delta\theta. \end{aligned} \tag{97}$$

Here we have used the equations in Appendix A.3 and $(h_{\underline{\mu\nu\rho}}\Gamma^{\underline{\mu\nu\rho}})^2 = 0$. To rearrange this equation into the desired form we have to express $\bar{\theta}\Gamma^{\underline{\mu}}\Gamma_{(5)}\delta\theta$ in terms of $\bar{\theta}\Gamma^{\underline{\mu}}\delta\theta$ and of $\bar{\theta}\Gamma_{\underline{\mu\nu}}\delta\theta$. For this purpose we rewrite eq.(27) as

$$\Gamma_{(5)}\theta = (1+\frac{1}{3}h_{\underline{\mu\nu\rho}}\Gamma^{\underline{\mu\nu\rho}})\theta. \tag{98}$$

Using this equation and equations in Appendix A.3 we find

$$(1+2k)^{\underline{\mu}}_{\underline{\nu}}\Gamma^{\underline{\nu}}\Gamma_{(5)}\theta = (1-2k)^{\underline{\mu}}_{\underline{\nu}}\Gamma^{\underline{\nu}}\theta + 2h^{\underline{\mu\nu\rho}}\Gamma_{\underline{\nu\rho}}\theta. \tag{99}$$

Substituting it into eq.(97) yields

$$\bar{\theta}\Gamma_{\underline{\mu\nu}}\delta\theta - k^{\underline{\rho}}_{\underline{\mu}}\bar{\theta}\Gamma_{\underline{\rho\nu}}\delta\theta - k^{\underline{\rho}}_{\underline{\nu}}\bar{\theta}\Gamma_{\underline{\mu\rho}}\delta\theta = 4h_{\underline{\mu\nu\rho}}\bar{\theta}\Gamma^{\underline{\rho}}\delta\theta. \tag{100}$$

An inspection shows that this is equivalent to the equation

$$\begin{aligned} \bar{\theta}\Gamma_{\underline{\mu\nu}}\delta\theta &= 4(1-\frac{2}{3}k^{\underline{\rho}}_{\underline{\sigma}}k^{\underline{\sigma}}_{\underline{\rho}})^{-1}(h_{\underline{\mu\nu\kappa}} + k^{\underline{\lambda}}_{\underline{\mu}}h_{\underline{\lambda\nu\kappa}} + k^{\underline{\lambda}}_{\underline{\nu}}h_{\underline{\mu\lambda\kappa}})\bar{\theta}\Gamma^{\underline{\kappa}}\delta\theta \\ &= 4h_{\underline{\mu\nu\sigma}}(1-2k)^{-1\underline{\sigma}}_{\underline{\rho}}\bar{\theta}\Gamma^{\underline{\rho}}\delta\theta, \end{aligned} \tag{101}$$

which is identical to the last condition of (24) providing eq.(28) holds.

C Matrix approximation of Dirichlet and Neumann modes

In this section we show that Dirichlet and Neumann modes are well approximated by $N \times N$ antisymmetric and symmetric matrices, respectively. We recall[29][30] that the Fourier modes on the torus (parametrized by $(\sigma^1, \sigma^2) \in [0,1)^2$) is approximated as

$$Y_A(\sigma) \equiv e^{2\pi i(A_1\sigma^1 + A_2\sigma^2)} \overset{N\to\infty}{\longleftarrow} T_A \equiv \frac{1}{\sqrt{N}}e^{-\frac{\pi i}{N}A_1A_2}V^{A_1}U^{A_2}, \tag{102}$$

where 't Hooft's twist matrices U and V have the following properties[31]

$$U^N = V^N = 1, \quad VU = e^{\frac{2\pi}{N}i} UV, \quad U^\dagger = U^{-1}, \quad V^\dagger = V^{-1}. \tag{103}$$

The phase factor of T_A is chosen so that we have

$$(T_A)^\dagger = T_{-A}. \tag{104}$$

We take the following representation for U, V

$$U = \begin{pmatrix} 1 & & & & 0 \\ & e^{\frac{2\pi}{N}i} & & & \\ & & \ddots & & \\ & & & \ddots & \\ 0 & & & & e^{\frac{2\pi i}{N}(N-1)} \end{pmatrix}, \qquad V = \begin{pmatrix} 0 & 1 & & & 0 \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ 1 & & & & 0 \end{pmatrix}, \tag{105}$$

with the properties $U^T = U$ and $V^T = V^{-1}$. Then we find the important relation

$$T_{(-A_1, A_2)} = (T_{(A_1, A_2)})^T. \tag{106}$$

This enables us to confirm that the following correspondence holds

$$\begin{aligned} Y_A^{(D)} &= \frac{-i}{\sqrt{2}}(Y_{(A_1,A_2)} - Y_{(-A_1,A_2)}) \quad \overset{N\to\infty}{\longleftarrow} \quad T_A^{(D)} \equiv \frac{-i}{\sqrt{2}}(T_A - (T_A)^T), \\ Y_A^{(N)} &= \frac{1}{\sqrt{2}}(Y_{(A_1,A_2)} + Y_{(-A_1,A_2)}) \quad \overset{N\to\infty}{\longleftarrow} \quad T_A^{(N)} \equiv \frac{1}{\sqrt{2}}(T_A + (T_A)^T). \end{aligned} \tag{107}$$

Because $T_A^{(D)}$ ($T_A^{(N)}$) are manifestly antisymmetric (symmetric), this implies the correspondence (52).

We note that, as far as the representation of SO(N) is concerned, the correspondence(52) is independent of the choice of twist matrices (U, V), because any twist matrices are unitary equivalent to those in eq.(105). For example one may choose U instead of V to define $e^{2\pi i \sigma_1}$. In this choice, (106) is replaced by

$$T_{(-A_1, A_2)} = (T_{(A_1, A_2)})^*. \tag{108}$$

In this convention, the generators associated with the Dirichlet modes $\sin(2\pi i n \sigma_1)\sin(2\pi i m \sigma_2)$ are anti-symmetric but those associated with another type $\sin(2\pi i n \sigma_1)\cos(2\pi i m \sigma_2)$ become symmetric. In this sense, our claim that the Dirichlet mode is described by the antisymmetric matrix depends on our specific choice of the basis. However, since these different choices are equivalent under a unitary transformation, the underlying algebraic structure remains the same.

D Matrix approximation of surface integration

In this section we examine whether the integration $\int_{\Sigma^{(2)}} d^2\sigma$ possesses the property of the trace in the restricted situation (54). We first note that, in the situation we consider, the integration on the cylinder equals to that on the torus, namely

$$\int_0^{1/2} d\sigma^1 \int_0^1 d\sigma^2 2F(\sigma) = \int_0^1 d\sigma^1 \int_0^1 d\sigma^2 F(\sigma),$$

where $F(\sigma^1, \sigma^2) = F(1-\sigma^1, \sigma^2)$ is a periodic function on the torus. Since the integration on the torus is well-approximated by the trace of $N \times N$ matrices, the same should be true for that on the cylinder as long as we consider the structure (54). For completeness we show that the cyclic identity

$$\int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} A\{B, C\} = \int_{\Sigma^{(2)}} d^2\sigma \sqrt{w} B\{C, A\} \tag{109}$$

holds in the following two cases.

(i)$\underline{A, B, C = \text{Dirichlet.}}$

$$\begin{aligned} L.H.S. &= \int d^2\sigma A \epsilon^{rs} \partial_r B \partial_s C \\ &= -\int d^2\sigma B \epsilon^{rs} \partial_r A \partial_s C + \int d\sigma^2 (AB\partial_2 C)|_{\sigma^1=0}^{\sigma^1=1/2} \\ &= R.H.S.. \end{aligned} \tag{110}$$

(ii) $\underline{A = \text{Dirichlet; } B, C = \text{Neumann.}}$

$$\begin{aligned} L.H.S. &= \int d^2\sigma \sqrt{w} B\{C, A\} + \int d\sigma^2 (AB\partial_2 C)|_{\sigma^1=0}^{\sigma^1=1/2} \\ &= R.H.S.. \end{aligned} \tag{111}$$

Thus we can approximate the integration by the trace in the situation which appears in the light-cone gauge formulation of the open supermembrane.

E General solutions of BPS equations

Let us start from the situation in ref.[20] where we have investigated the BPS conditions for the closed toroidal supermembrane in the target space which is toroidally compactified, i.e., $X^a \sim X^a + 2\pi R^a$ $(a = 1, \ldots, 9)$ and $X^- \sim X^- + 2\pi R$. In general the embedding functions and their conjugate momenta are expanded as

$$\begin{aligned} X^- &= -Ht + 2\pi R n_r \sigma^r + \hat{X}^-(\sigma^1, \sigma^2, t) \\ X^a &= \frac{m^a}{R^a} t + 2\pi R^a n_r^a \sigma^r + \hat{X}^a(\sigma^1, \sigma^2, t), \\ P^a &\equiv \partial_t X^a = \frac{m^a}{R^a} + \hat{P}^a(\sigma^1, \sigma^2, t), \end{aligned} \tag{112}$$

where we have used the rescaled time $t \equiv \frac{R}{m}\tau$ and the symbols with hat denote the contributions from the oscillating modes on $\Sigma^{(2)} \approx T^2$.

We have also introduced a nine-dimensional orthonormal basis $(e_a^{(9)}, e_a^{(i)})$ $(i = 1, \ldots, 8)$ such that $P_0^+ z^a - P_0^c z^{ca} \propto e^{(9)a}$, where z^a and z^{ca} are longitudinal and transverse membrane charges, respectively. We denote the components of a nine-dimensional vector V^a as

$$V^9 = e_a^{(9)} V^a, \quad V^i = e_a^{(i)} V^a.$$

For simplicity we set $\hat{\theta} = 0$. Extension to the case of nonzero $\hat{\theta}$ can be carried out if we use the prescription explained in sec.5 of ref.[20].

In this setup we have shown that the configurations with 1/4 SUSY must satisfy the BPS conditions

$$\begin{aligned} \hat{P}^9 &= 0, && (113)\\ \hat{P}^i &= \pm\left(\left\{X^9, X^i\right\} + z^{9i}\right), && (114)\\ 0 &= \left\{X^i, X^j\right\} + z^{ij}, && (115)\end{aligned}$$

as well as the constraints

$$\begin{aligned} 0 &= \varphi(\sigma) = \nabla^a \hat{P}^a + \{\hat{X}^a, \hat{P}^a\}, && (116)\\ 0 &= \varphi_r = m n_r + m^a n_r^a + \frac{1}{2\pi}\int d^2\sigma \hat{P}^a \partial_r \hat{X}^a, && (117)\\ 0 &= P_0^+ z^i - P_0^c z^{ci}, && (118)\end{aligned}$$

where $\nabla^a \equiv 2\pi R^a (n_1^a \partial_2 - n_2^a \partial_1)$.

In the following we investigate what the general solutions of eqs.(113- 118) would look like. We note that, among these equations, the consistency condition (118) is redundant because it automatically holds as a consequence of eqs.(113),(115),(116) and (117):

$$\begin{aligned} P_0^+ z^i - P_0^c z^{ci} &= \int d^2\sigma \hat{P}^c \nabla^i \hat{X}^c \\ &= \int d^2\sigma \hat{P}^c (\nabla^i \hat{X}^c - \nabla^c \hat{X}^i + \{\hat{X}^i, \hat{X}^c\}) \\ &= \int d^2\sigma \hat{P}^j (\{X^i, X^j\} + z^{ij}) \\ &= 0. && (119)\end{aligned}$$

It is trivial to solve the first BPS condition (113). The general solution is,

$$X^9 = P_0^9 t + 2\pi \tilde{R}_r^9 \sigma^r + \xi(\sigma^1, \sigma^2), \tag{120}$$

where $\tilde{R}_r^9 \equiv \sum_{a=1}^9 e_a^{(9)} R^a n_r^a$. Assuming that ξ is sufficiently small, a local APD gauge transformation further reduces this to the following form

$$X^9 = P_0^9 t + 2\pi \tilde{R}_r^9 \sigma^r + \xi(\tilde{R}_r^9 \sigma^r). \tag{121}$$

In general, nonlinear partial differential equation (115) is hard to solve. Here we consider physically interesting situation where there exist nonvanishing windings $\tilde{R}_r^i \equiv \sum_{a=1}^9 e_a^{(i)} R^a n_r^a$

and the oscillating modes $\hat{X}^i$ are infinitesimally small. In this case the equation reduces to the linear partial differential equation,

$$\nabla^i \hat{X}^j - \nabla^j \hat{X}^i = 0, \quad \nabla^i \equiv e_a^{(i)} \nabla^a = 2\pi(\tilde{R}_1^i \partial_2 - \tilde{R}_2^i \partial_1). \tag{122}$$

The linearlized version can be straightfowardly solved. The general solution to the equation for one pair (say (i,j)) is given as follows,

$$\begin{aligned} \hat{X}^i &= \nabla^i \epsilon_{ij}(\sigma^1, \sigma^2, t) + \eta^i(\sigma^1, \sigma^2, t), \\ \hat{X}^j &= \nabla^j \epsilon_{ij}(\sigma^1, \sigma^2, t) + \eta^j(\sigma^1, \sigma^2, t), \end{aligned} \tag{123}$$

where η^i, η^j satisfy

$$\nabla^j \eta^i = \nabla^i \eta^j = 0. \tag{124}$$

The analysis has to be made separately for the two cases, $z^{ij} = 0$ and $z^{ij} \neq 0$. But the final answer can be written in the same way.

Suppose that we need to consider T^d with $d > 2$, we have to study the consistency condition among the solutions for every pair. It seems that we need to make classification of the solution into two cases.

(a) Every z^{ij} vanishes. In this case, we can factorize $\tilde{R}_r^i$ as $\tilde{R}_r^i = k^i \tilde{R}_r$ and we can take $\eta^i \neq 0$. This η is the string excitation in the double dimensional reduction. ϵs are constrained to be the same for every pair, namely $\epsilon^{ij} = \epsilon$. Thus we find

$$X^i = P_0^i t + 2\pi k^i \tilde{R}_r \sigma^r + \nabla^i \epsilon(\sigma^1, \sigma^2, t) + \eta^i(\tilde{R}_r \sigma^r, t). \tag{125}$$

(b) There are some nonvanishing z^{ij}. In this case, we need to put every η to vanish. As before every ϵ are the same $\epsilon^{ij} = \epsilon$. The result is

$$X^i = P_0^i t + 2\pi \tilde{R}_r^i \sigma^r + \nabla^i \epsilon(\sigma^1, \sigma^2, t). \tag{126}$$

Next we solve the linearized version of the constraint (116), namely, $\nabla^i \hat{P}^i = 0$. By substisuting (125) or (126) into this equation, we find that ϵ is time-independent, apart from the part which can be absorbed into η^i.

The equation (114) defines the time evolution of remaining unknown quantities ϵ and η. It is easier to study the case (b). The result of substituting (121) and (126) into (114) is given by $0 = \nabla^i[\nabla^9 \epsilon(\sigma^1, \sigma^2) - \xi(\tilde{R}_r^9 \sigma^r)]$. From this we can easily see

$$\nabla^9 \epsilon(\sigma^1, \sigma^2) - \xi(\tilde{R}_r^9 \sigma^r) = 0. \tag{127}$$

In order for this to hold we have to set

$$\xi = 0, \qquad \epsilon = \epsilon(\tilde{R}_r^9 \sigma^r). \tag{128}$$

We can absorb this ϵ by an infinitesimal APD. The final result is

$$X^9 = P_0^9 t + 2\pi \tilde{R}_r^9 \sigma^r, \quad X^i = P_0^i t + 2\pi \tilde{R}_r^i \sigma^r. \tag{129}$$

This is nothing but the BPS configuration with 1/2 SUSY.

Let us now examine the case (a). By substituting (121) and (125) into (114), we find

$$(\partial_t \mp \nabla^9)\eta^i(\tilde{R}_r\sigma^r, t) = \pm k^i \nabla(\nabla^9 \epsilon(\sigma^1, \sigma^2) - \xi(\tilde{R}^9_r\sigma^r)), \tag{130}$$

where $\nabla \equiv 2\pi(\tilde{R}_1\partial_2 - \tilde{R}_2\partial_1)$. This implies the following equations

$$\begin{aligned} (\frac{m}{R}\partial_t \mp \nabla^9)\eta^i &= 0, &(131)\\ \epsilon(\sigma^1, \sigma^2) &= \epsilon^{(1)}(\tilde{R}_r\sigma^r) + \epsilon^{(2)}(\tilde{R}^9_r\sigma^r), &(132)\\ \xi(\tilde{R}^9_r\sigma^r) &= 0. &(133)\end{aligned}$$

Plugging these back into eqs.(121)and (125) and performing an appropriate APD transformation we find

$$\begin{aligned} X^9 &= P^9_0 t + 2\pi\tilde{R}^9_r\sigma^r, \\ X^i &= P^i_0 t + 2\pi k^i\tilde{R}_r\sigma^r + \eta^i(\tilde{R}_r\sigma^r). \end{aligned} \tag{134}$$

We note that, if the ratio $\tilde{R}_1/\tilde{R}_2$ is irrational, η^i in eq.(134) become constant and the solution reduces to that with 1/2 SUSY. Thus, in order to obtain the nontrivial BPS configuration with 1/4 SUSY, we have to take this ratio to be rational. In this case, by using an appropriate $SL(2, \mathbf{Z})$ transformation

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \sigma'_1 \\ \sigma'_2 \end{pmatrix}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbf{Z}), \tag{135}$$

we can rewrite eq.(134) in the form

$$\begin{aligned} X^9 &= P^9_0 t + 2\pi\tilde{R}'^9_r\sigma^r, \\ X^i &= P^i_0 t + 2\pi k^i\tilde{R}'\sigma^1 + \hat{X}^i(\sigma^1, t), \end{aligned} \tag{136}$$

where $\hat{X}^i$ satisfy the equation

$$\partial_t\hat{X}^i = \mp 2\pi\tilde{R}^9_2\partial_1\hat{X}^i. \tag{137}$$

This final result is nothing but the extended version of the "stringy" configuration given in ref.[20], with the only extension being the winding of the σ^1-cycle in the X^9-direction.

The extension of the above results to the open supermembrane is straightforward. Note that we have $2\pi(\tilde{R}^9_1, \tilde{R}^9_2) = (b, 0)$ and σ^1 in the above analysis corresponds to $2\sigma^1$ in subsec. IV A. We only have to inspect what constraints are imposed by the bondary conditions. It turns out that the resulting configuration is given by eqs.(83) and (84).

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